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D1 – Mathematics
problems based on charpits method
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Q. 1:– Solve by charpits method to the equation $p(1+q^2)=q(z-a)$

Solution :– We have $F(x, y, z, p, q)$

$$= p(1+q^2) - q(z-a) = 0$$

$$\frac{\partial F}{\partial x} = 0, \quad \frac{\partial F}{\partial y} = 0 \quad \& \quad \frac{\partial F}{\partial z} = -q$$

$$\frac{\partial F}{\partial p} = 1+q^2, \quad \frac{\partial F}{\partial q} = 2pq - (z-a)$$

Hence charpits equation are

$$\frac{dp}{0+p(-q)} = \frac{dq}{0+q(-q)} = \frac{dz}{-p(1+q^2)-q(2pq-z+a)}$$

$$= \frac{dx}{-(1+q^2)} = \frac{dy}{-2pq+(z-a)}$$

$$\Rightarrow \frac{dp}{-pq} = \frac{dq}{-q^2} = \frac{dz}{-(3pq^2+p+q(a-z))} = \frac{dx}{-(q^2+1)}$$

$$= \frac{dy}{-(2pq-z+a)}$$

$$\Rightarrow \frac{dp}{pq} = \frac{dq}{q^2} = \frac{dz}{3pq^2+p+q(a-z)}$$

$$= \frac{dx}{q^2+1} = \frac{dy}{2pq-z+a}$$

From the first two ratio's, we get

$$\frac{dp}{p} = \frac{dq}{q}$$

Integrating we get $\log q = \log p + \log k$
$= \log pk$

putting $q = kp$ in the given equation,

$p(1+k^2p^2) = kp(z-a) \Rightarrow 1+k^2p^2 = k(z-a)$

$\Rightarrow k^2p^2 = k(z-a)-1 \Rightarrow p = \frac{\sqrt{k(z-a)-1}}{k}$

$\therefore q = kp = \sqrt{k(z-a)-1}$

Now $dz = p\,dx + q\,dy$

$\Rightarrow dz = \frac{\sqrt{k(z-a)-1}}{k}\,dx + \sqrt{k(z-a)-1}\,dy$

$\Rightarrow dz = \sqrt{k(z-a)-1}\left(\frac{1}{k}dx + dy\right)$

$\Rightarrow \frac{k\,dz}{\sqrt{k(z-a)-1}} = dx + k\,dy$

Integrating, we get

$4k(z-a) - 4 = (x+ky+b)^2 \Rightarrow 4k(z-a) = (x+ky+b^2)+4$